

Metal Whiskers:

Failure Modes and Mitigation Strategies

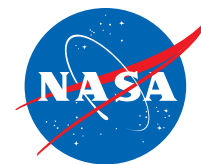
Jay Brusse / Perot Systems

Dr. Henning Leidecker / NASA Goddard

Lyudmyla Panashchenko / Univ. of MD-CALCE Graduate Student



<http://nepp.nasa.gov/whisker>



Outline

- A Brief History of Metal Whiskers
- System Failure Modes Caused by Metal Whiskers
- A Few Mitigation Strategies to Reduce Harm From Metal Whiskers

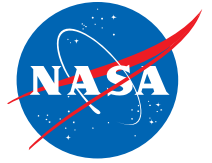


**Zinc Whiskers on
Hot Dip Galvanized Steel Pipe**

**Cover Photo:
Tin whiskers on Tin-Plated
Beryllium Copper PCB Card Rails**

- ***NO WHISKER GROWTH THEORY TO BE DISCUSSED!!!***

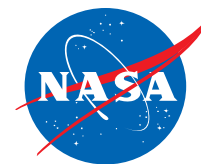
What are Tin or Zinc or Cadmium Whiskers?



- Hair-like structures made of a single grain, or only a few grains, that sometimes erupt from a metal. Coatings of Tin, Zinc and Cadmium are especially able to develop whiskers; but, whiskers have been seen on Gold, Silver, Lead, and other metals too
- Growth occurs over time by accretion of metal ions at the base NOT the tip
- **LENGTH:** Log-normally distributed
Rarely up to 10 mm or more
(Typically ~1mm or less)
- **THICKNESS:** Range 0.006 to >10 μm
(Typical ~ 1 μm)
- **Fundamental theories for growth mechanism DO NOT enable prediction of the time-dependence of whisker density, whisker lengths or thicknesses**
 - To be useful a theory should identify what we must control to make confident predictions
 - Such a theory has remained elusive



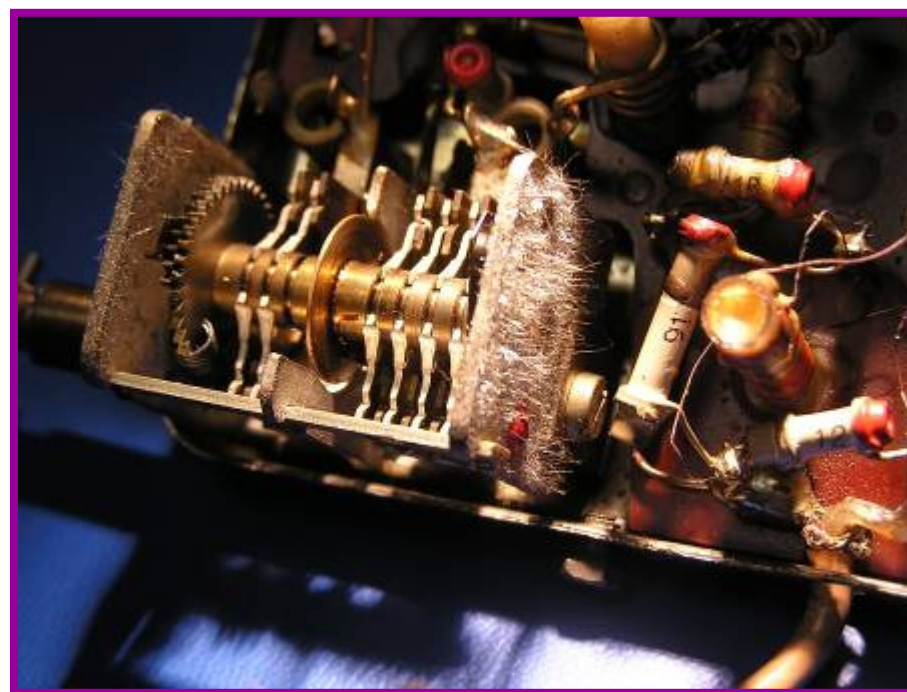
*Tin Whiskers on Tin-Plated
Ceramic Chip Capacitor*



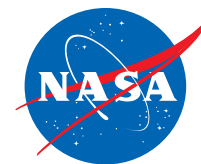
Metal Whiskers

“The Early Years”

- **1946:**
H. Cobb (Aircraft Radio Corp.) publishes earliest “known” account of CADMIUM whiskers inducing electrical shorting between plates of air capacitors used in military equipment. These events occurred during World War II (~1942 – 1943)
- **1952:**
Since Cadmium coatings resulted in shorting, Tin and Zinc were used instead. But then K.G. Compton, A. Mendizza, and S.M. Arnold (Bell Labs) reported shorting caused by whiskers from these coatings too!



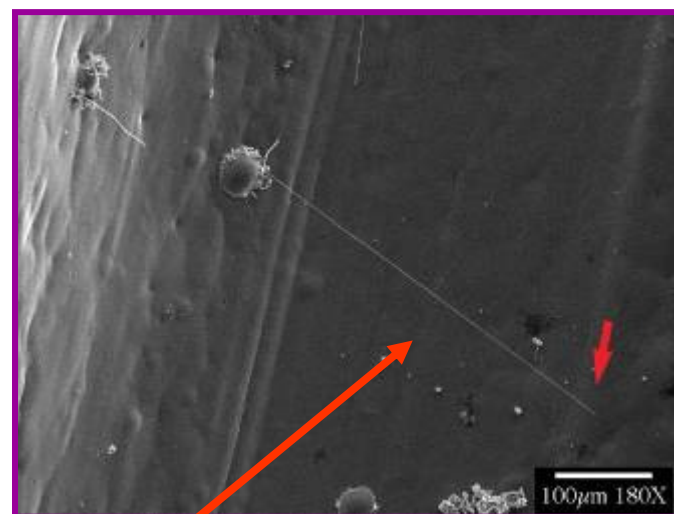
*Tin Whiskers on 1960's Era
Variable Air Capacitor*



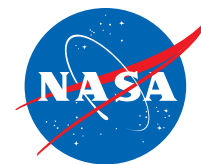
Whisker Resistant Metal Coatings

“The Quest”

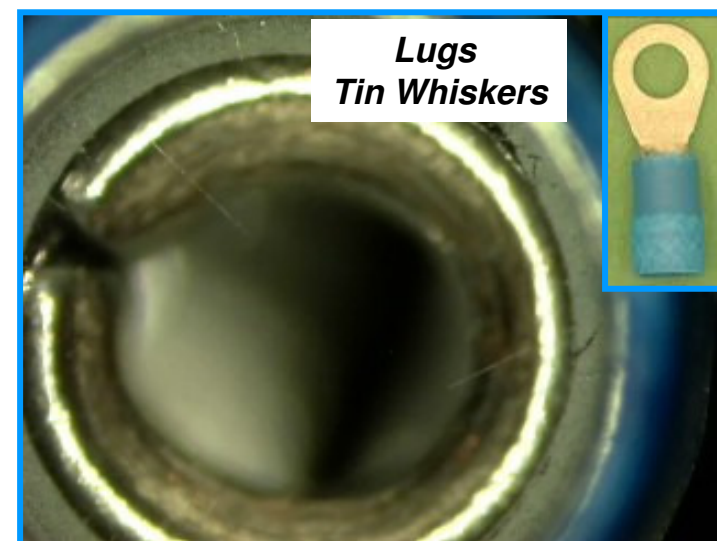
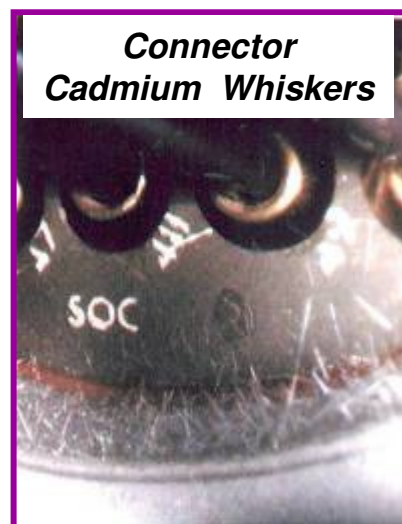
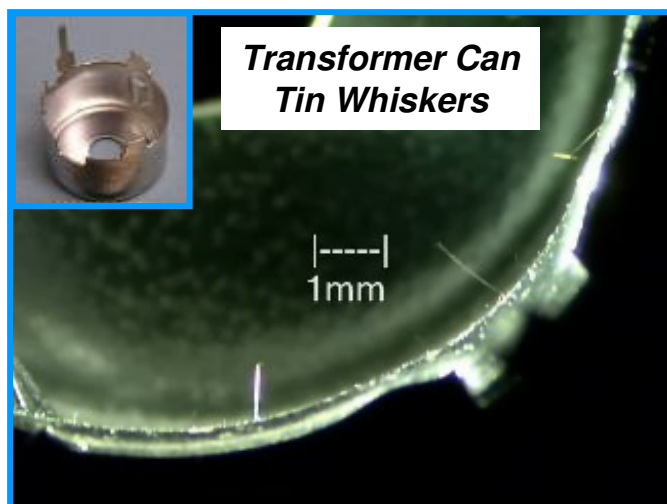
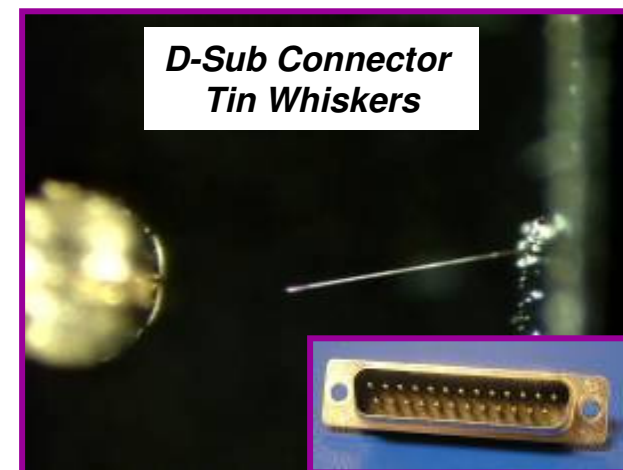
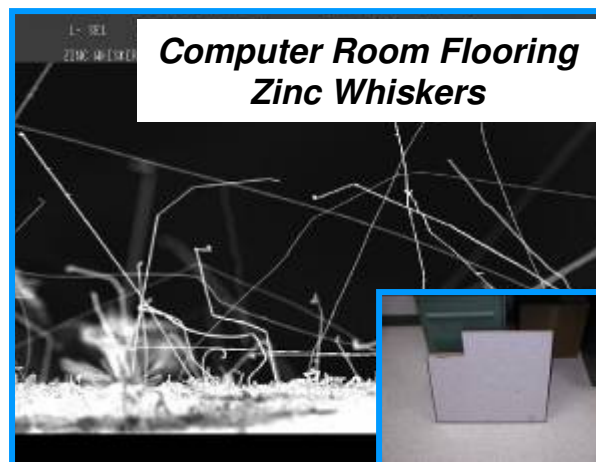
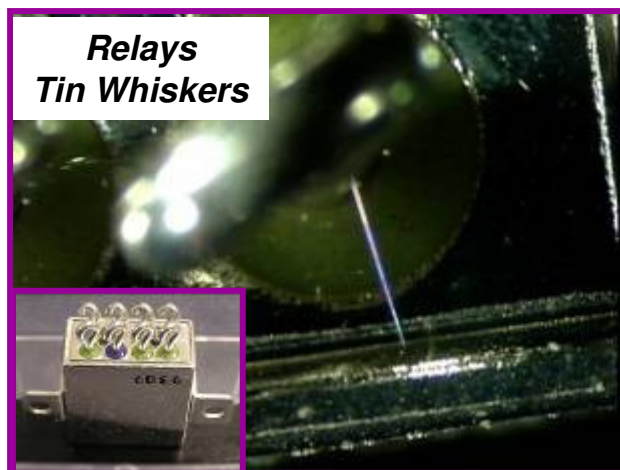
- **1950s and 60’s:**
Bell Labs worked through the periodic table, to determine whether addition of some element to a Tin coating would “quench” whiskering
 - *Adding 0.5 - 1% (by weight) of lead (Pb) works*
 - *Some additives seem to enhance whiskering*
- **Since 1990s:**
Most MIL specs require adding Pb to any tin coatings used around electronics.
 - Concentration is usually named as 2% to 3% Pb by weight for “margin”
- **What additives quench zinc and cadmium whiskers?**
 - We don’t know, but certainly NOT chromate conversion finishes!



***Zinc Whiskers Growing from
Zinc-Plated Yellow Chromate
Steel Bus Rail***

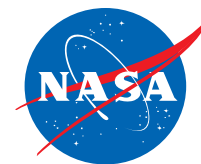


Metal Whiskers on Components



December 5, 2007

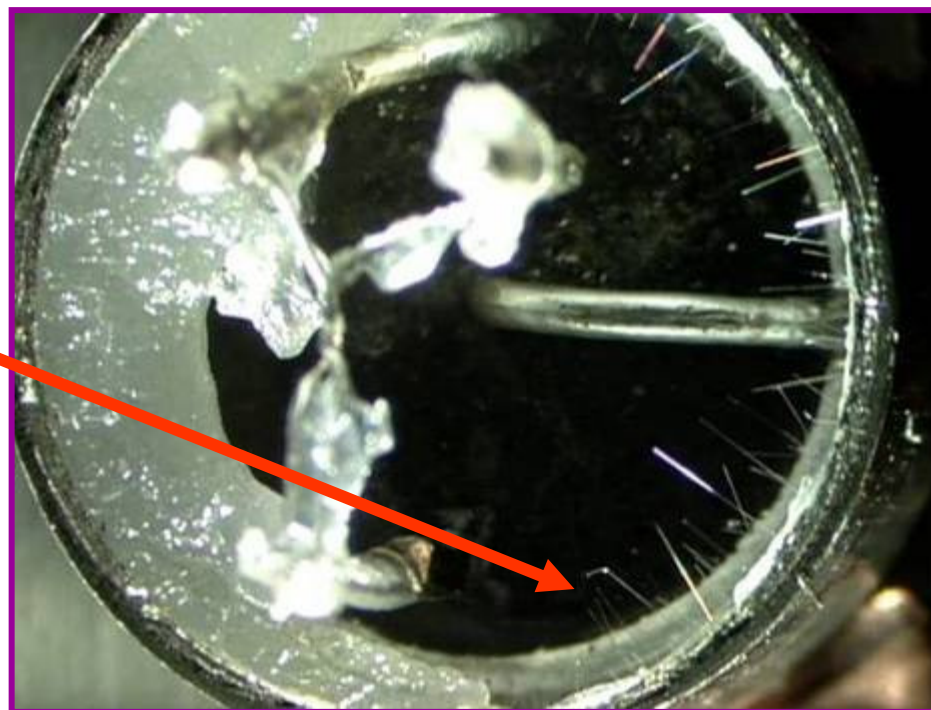
Metal Whiskers:
Failure Modes and Mitigation Strategies



Guess What's Lurking Inside?



1960's Vintage Transistor

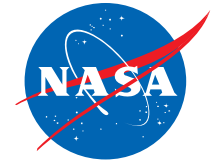


Transistor Package is Tin-Plated Inside.

**Many Vintage Radio Malfunctions Have Been
Attributed to Whiskers Shunting Case to Terminals**

<http://www.vintage-radio.net/forum/showthread.php?t=5058>

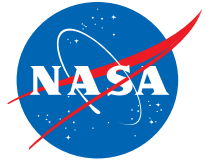
2006- NASA GSFC Presented A Partial History of Documented Metal Whisker Problems



http://nepp.nasa.gov/whisker/reference/tech_papers/2006-Leidecker-Tin-Whisker-Failures.pdf

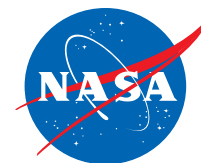
Year**	Application	Industry	Failure Cause	Whiskers on?
1946	Military	Military	Cadmium Whiskers	Capacitor plates
1948	Telecom Equipment			
1954	Telecom Equipment			
1959	Telecom Equipment			

These are ~10% of the Problems We Know About



*“There is a name for those who suppose that doing the same thing will produce different results. That name is ‘**Idiot**’.”*

- Albert Einstein



Basic Whisker Failure Modes

Electrical Short Circuits

- Continuous short if
- Intermittent short if

$$\begin{aligned} I_{whisker} &< I_{melt} \\ I_{whisker} &> I_{melt} \end{aligned}$$

Debris/Contamination

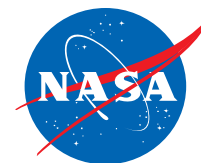
- Interfere with Sensitive Optics or MEMS
- Produce Shorts in Areas REMOTE From Whisker Origins
(Zinc Whiskers on raised flooring are a PRIME Example)

METAL VAPOR ARC

- If $I_{whisker} \gg I_{melt}$ Whisker Can Vaporize into a Metal Gas
- If EMF is sufficiently high, then the metal gas can be ionized into a conductive PLASMA of Metal Ions
- Plasma Can Form an Arc Capable of Carrying **HUNDREDS OF AMPS!** Depends on arc gap length, voltage, current, pressure, etc.

December 5, 2007





Metal Whisker Melting Current -- Pt. 1

$$I_{melt,vac} = \left[\frac{2\sqrt{L_z T_0}}{R_0} \right] \cos^{-1} \left(\frac{T_0}{T_{melt}} \right)$$

See Backup Slides for Derivation

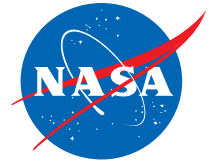
- Where $L_z \sim 2.45 \cdot 10^{-8} (V/K)^2$ is the Lorenz number, T_{melt} = melting temperature, T_0 = ambient temperature, R_0 = whisker resistance at ambient

Material	T_{melt}	$I_{melt, vac}$	$V_{melt} = R_0 * I_{melt, vac}$
Tin	505.1K	87.5 mV / R_0	88 mV
Cadmium	594.2K	97.1 mV / R_0	97 mV
Zinc	692.7K	104.4 mV / R_0	104 mV

*If $V_{whisker} > V_{melt}$
Then the Whisker will Fuse Open*

But there is MORE to this story

Metal Whisker Melting Current -- Pt. 2



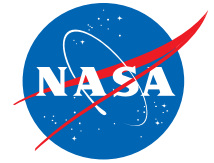
- Metal whiskers are also coated with electrically insulating films
 - Tin oxides, Zinc oxides, moisture films, et al
- Direct **MECHANICAL** contact by the whisker to another conductor does NOT guarantee **ELECTRICAL** contact
 - For Electrical Contact, the potential difference must exceed “dielectric breakdown” of the insulating films
 - For tin and zinc whiskers, independent groups have confirmed the oxide film breakdown ranges from $\sim 0.2V$ to $\sim 45V$



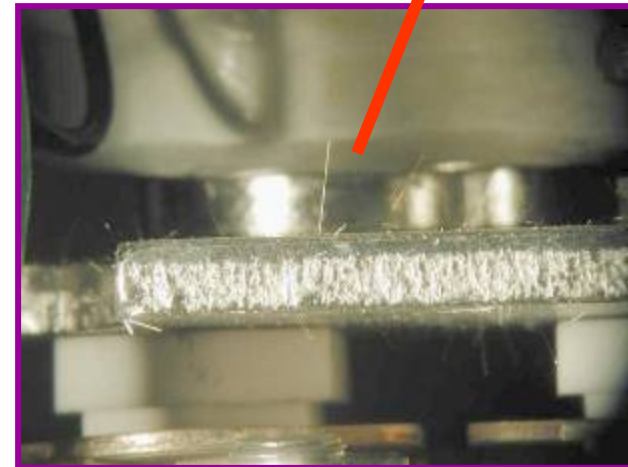
Electrical Current Flow through this Tin Whisker did not occur until $V_{\text{applied}} \sim 3 V$ at which point the Whisker promptly melted

See Backup Slides for a Suggested Circuit for Measuring Metal Whisker Resistance

Sustained Metal Vapor Arcing

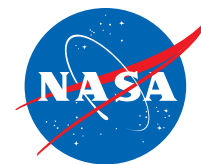


- Gaps shorter than a few tens of microns can support a sustained arc at potential differences of 12 to 14 volts, and currents of 0.1 to 0.3 amperes.
 - See “Electrical Contacts” by Paul G. Slade, part three
- However, as the arc gap increases, the EMF needed to sustain the arc increases, as does the current.
 - GSFC testing of FM08 style fuses with metal filaments ~5 mm long finds ~ 75 volts at more than 30 amperes is needed to generate a sustained arc across this arc gap when $P \sim 1$ torr



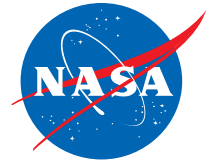
***Tin Whiskers Growing on Armature
Of Relay Produced Metal Vapor Arc***

G. Davy, "[Relay Failure Caused by Tin Whiskers](http://nepp.nasa.gov/whisker/reference/tech_papers/davy2002-relay-failure-caused-by-tin-whiskers.pdf)", Northrop Grumman, Technical Article, October 2002
http://nepp.nasa.gov/whisker/reference/tech_papers/davy2002-relay-failure-caused-by-tin-whiskers.pdf



How do People with “Whiskers” Cope?

My Whisker *“Stress Relaxation Theory”*



Man with “Facial Whiskers” Does YOGA!



Men with “Metal Whiskers” Find Innovative Ways to Relieve Stress

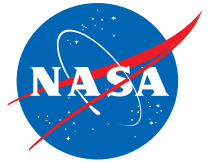


December 5, 2007

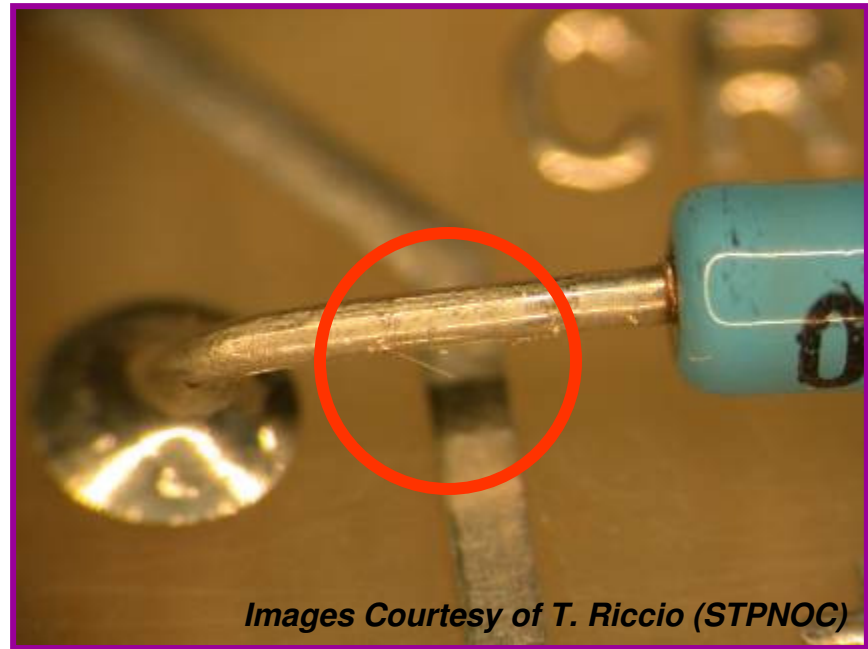
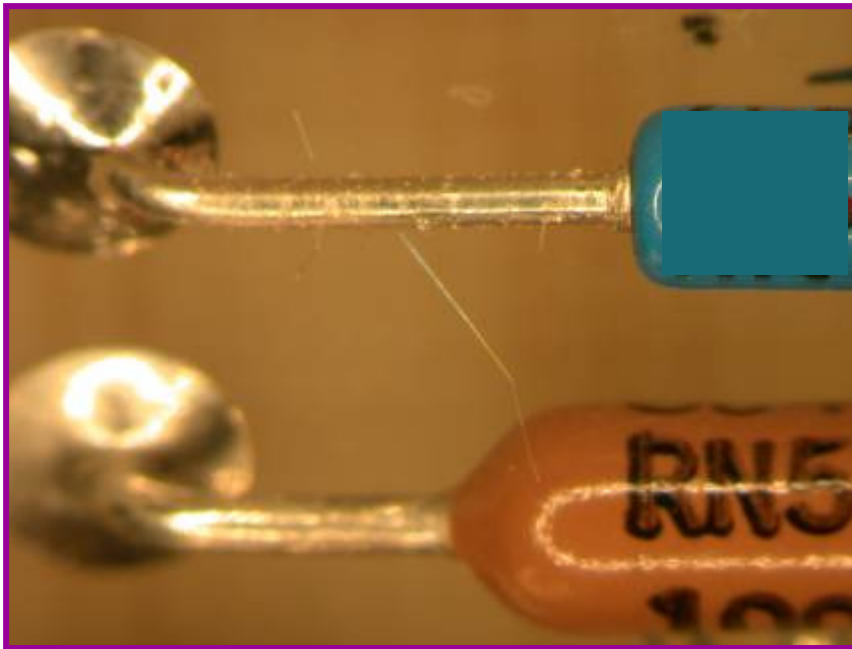
Metal Whiskers:
Failure Modes and Mitigation Strategies

15

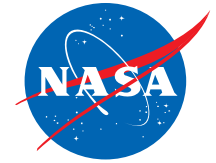
A Case for Whisker Mitigation Strategies?



Tin Whiskers on Tin-Plated Axial Leaded Diodes



- PWB and components were **NOT Conformal Coated**
- Diode Leads were **NOT Hot Solder Dipped**



Three Whisker Mitigation Strategies

Mitigation – to make less severe or painful

Merriam-Webster Dictionary

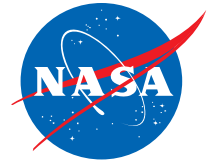
Risk “Mitigation” $\neq$ Risk “Elimination”

- Avoid Use of Whisker Prone Surface Finishes
 - Perform independent materials composition analysis
 - “*Trust, But VERIFY!*” using X-ray Fluorescence (XRF), Energy Dispersive X-ray Spectroscopy (EDS), et al
- Conformal Coat
 - Can slow whisker growth
 - Can block whiskers from electrically shunting distant conductors
- Remove/Replace Tin Finishes When Practical
 - Hot Solder Dip using lead-tin (Pb-Sn) solders
 - “First Do No Harm” Principle

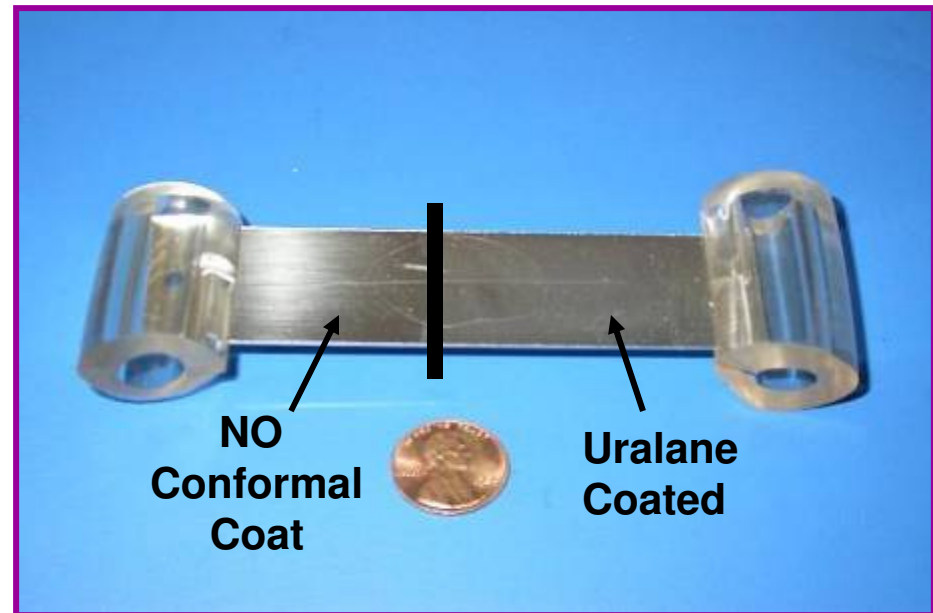
NASA Goddard Whisker Mitigation Study

Conformal Coat (Uralane 5750* Polyurethane)

~9 Years of Office Ambient Storage



- **Specimen:**
 - 1" x 4" x 1/16" Brass 260
 - Tin-Plated 200 microinches
 - Intentional scratches created after plating
- **Conformal Coating:**
 - Uralane 5750 on 1/2 of sample
 - Nominal Thickness = 2 mils
- **Storage Conditions:**
 - Office Ambient ~ 9 years

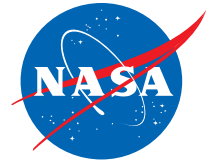


* *Uralane™ 5750 now known as Aralane™ 5750*

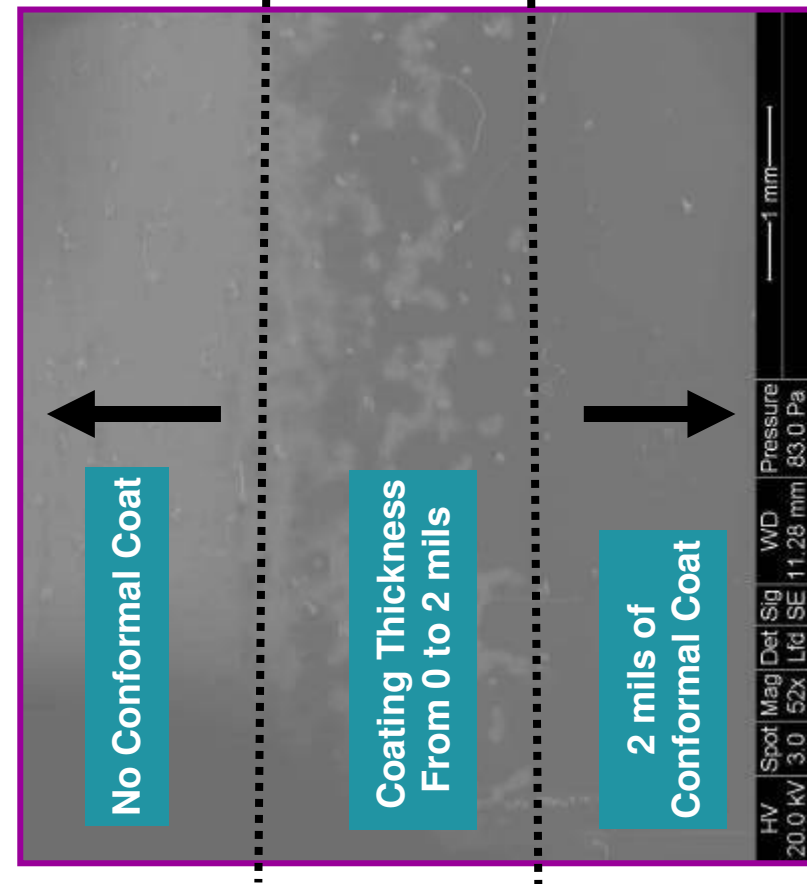
NASA Goddard Whisker Mitigation Study

Conformal Coat (Uralane 5750* Polyurethane)

~9 Years of Office Ambient Storage



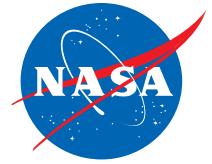
- Coating Thickness Can Vary Depending on Process Parameters
- NASA Test Coupons had a “transition” region ~2 mm wide where the conformal coating thickness was variable between 0 and 2 mils
- *One must understand their own processes to ensure the coating thickness is sufficient everywhere you intend it to be!!*



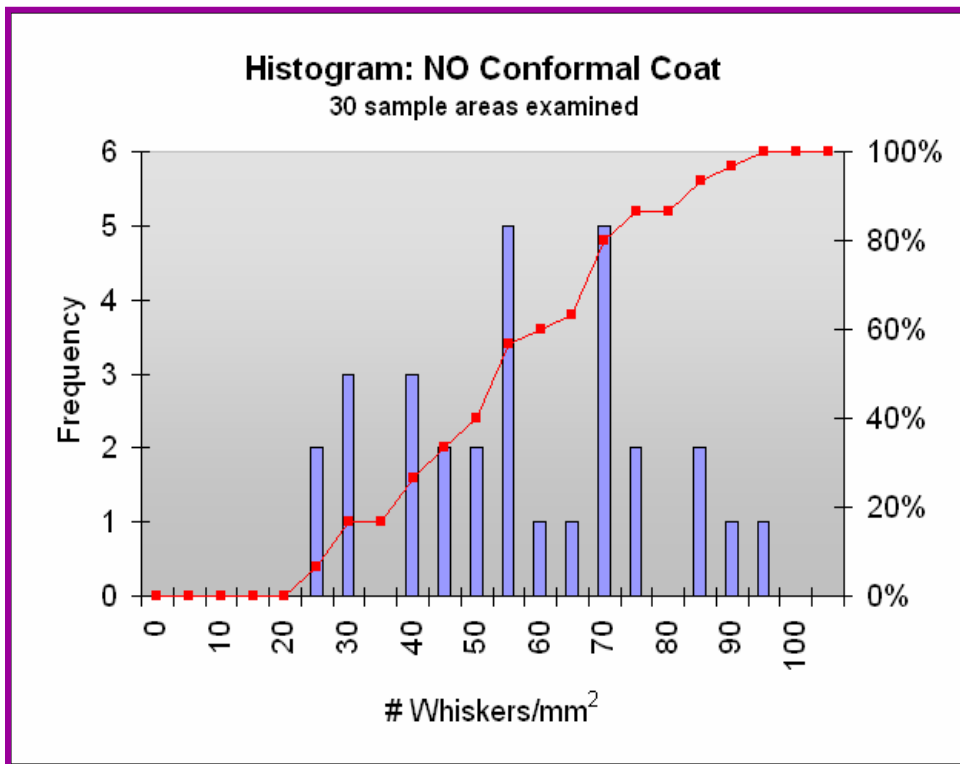
**NASA GSFC Conformal Coat
Tin Whisker Test Coupon**

* Uralane™ 5750 now known as Aralane™ 5750

Control Areas – No Conformal Coat 9-Years of Office Ambient Storage



- **Control Areas Grew Whiskers Abundantly**
 - Avg: 55 ± 19.6 whiskers / mm^2
 - Range: 23 to 95 whiskers / mm^2

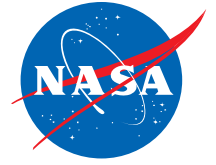


December 5, 2007

Metal Whiskers:
Failure Modes and Mitigation Strategies

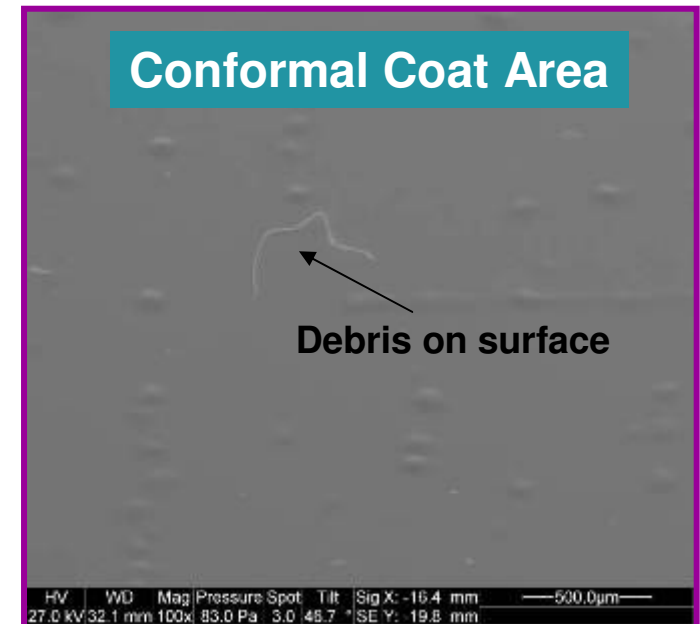
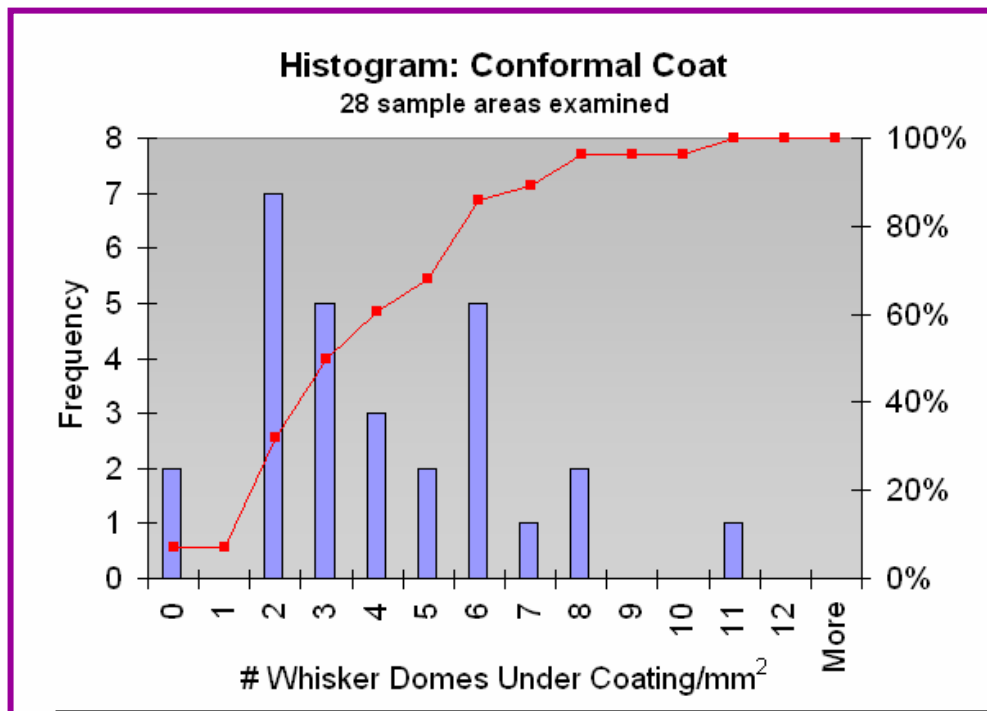
20

Uralane 5750 – 2 Mils Thick 9-Years of Office Ambient Storage



- Conformal Coated Areas Grew Whiskers Too
 - *To date ALL whiskers are contained beneath the coating that is 2 mils thick*
 - *With SEM we cannot see into the coating. Instead we see “domes” caused by whiskers that lift coating slightly*
 - Avg: 3.4 ± 2.6 domes / mm²
 - Range: 0 to 10.6 domes / mm²

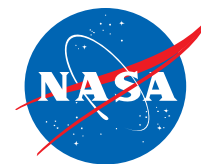
We suspect we are only counting “thick” whiskers in this statistic because the “thin” ones mechanically buckle before they can lift the coating enough to produce visible “domes”



December 5, 2007

Metal Whiskers:
Failure Modes and Mitigation Strategies

Whisker Puncture vs. Coating Thickness



~2 mils of Uralane 5750



Whiskers completely contained
BENEATH the coating
With nominal thickness of 2 mils

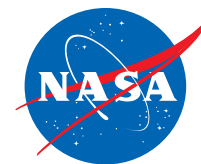
HV	WD	Mag	Pressure	Det	Spot	X: 17.6 mm	Tilt	200.0µm
800.0 V	26.7 mm	300x	---	ETD	3.0	Y: 6.8 mm	61.4 °	

Decreasing Coating Thickness



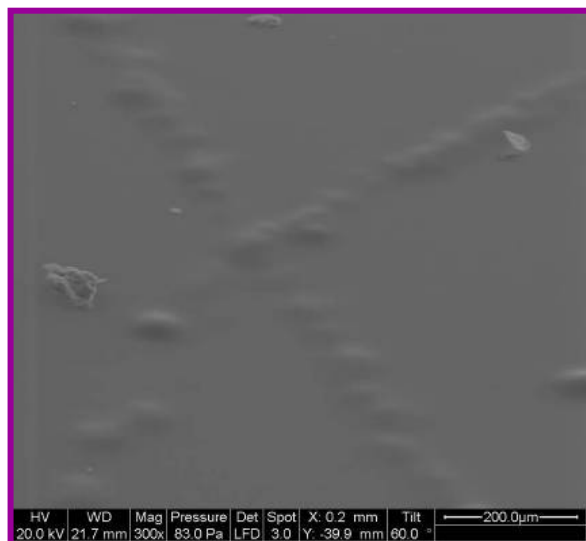
Whiskers punch through
in this region where
Coating thickness $\leq$ ~0.2 mils

HV	WD	Mag	Pressure	Det	Spot	X: 18.5 mm	Tilt	200.0µm
800.0 V	27.5 mm	300x	---	ETD	3.0	Y: 6.5 mm	61.4 °	



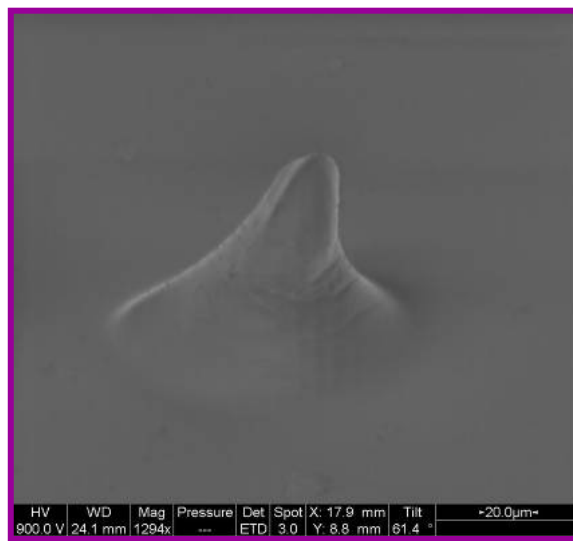
Uralane 5750 Conformal Coat - 9-Years of Office Ambient Storage

**2 Mils Uralane =
Very Effective**



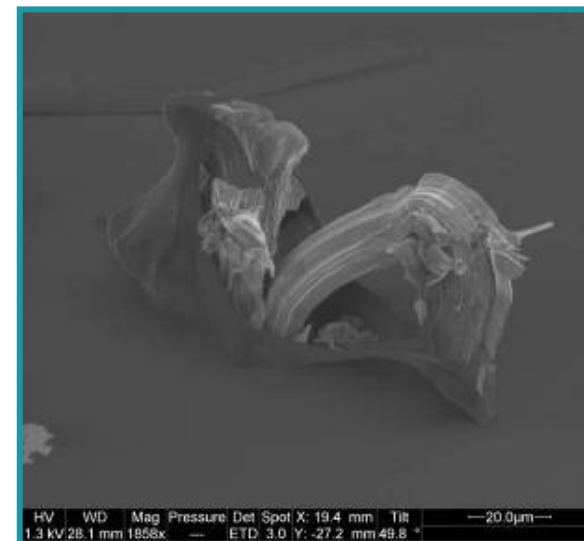
**Whiskers Completely
Entrapped Under the
Coating → Euler Buckling**

**~0.5 Mils Uralane =
Less Effective**

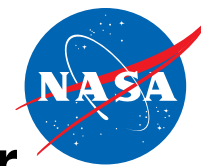


**Whisker “Lifting” Coating
into Shape of Circus Tent,
But Not Yet Penetrating**

**~0.1 Mils Uralane =
Not Effective**



**Whiskers Breaking
Through
“Thin” Coating**



Euler Buckling

Axial Force Required to Buckle a Metal Whisker

$$F_B = \frac{\pi^2 EI}{(KL)^2} \approx \left(\frac{\pi^3 \cdot E}{32} \right) \left(\frac{d^4}{L^2} \right)$$

E = Young's Modulus of whisker material,

I = Area Moment of Inertia,

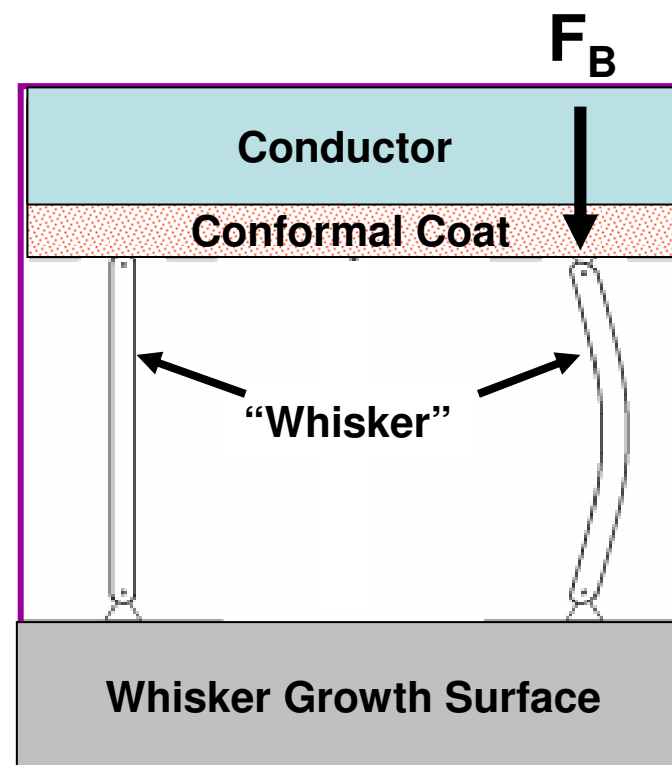
(e.g. $I = \pi d^4 / 64$ for circular cross section)

L = Length of whisker,

K = Column Effective Length Factor

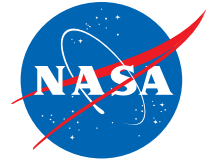
K = 0.5 for whisker fixed at both ends

K = 0.7 for fixed at one end, pinned at other

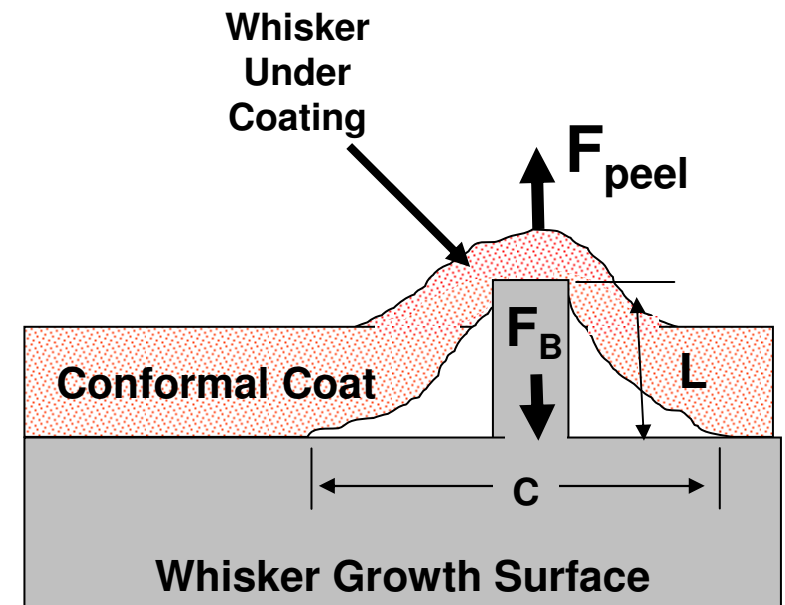


Whiskers Lift and Peel Conformal Coat Until Whisker Buckles OR Coating Fails

(F_{peel} vs. F_{Buckle})



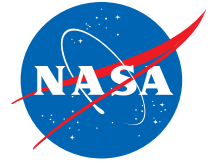
- As whisker first emerges it is short and stiff thus $F_B > F_{\text{peel}}$ and whisker begins to lift the coating forming a “circus tent” with height L = length of whisker;
- “Tent” joins the surface at a circle of circumference $C \sim 2\pi QL$,
 - Q describes the details of tent-like shape
- To peel conformal coating up and away from the surface, one needs to apply a force (F_{peel}) proportional to the circumference:
 - $F_{\text{peel}} = \Phi * C = 2\pi Q \Phi L$
 Φ = peel strength of material which describes the adhesion of the coating to the tin, and the effect of the separation angle. It also depends on the rate at which the coating is peeled away.



Uralane 5750 has better self-cohesion than adhesion to a tin surface

Additional Analysis Pending

Will Whiskers Buckle Before Puncturing a Distant Coated Surface?



- The displacement of the conformal coat due to a whisker pushing against the coating is:

$$D = \left(\frac{1 - \nu^2}{E_{coat}} \right) \left(\frac{F_B}{d} \right) \approx \left(\frac{\pi^3}{32} \right) (1 - \nu^2) \left(\frac{E_W}{E_{coat}} \right) \left(\frac{d^3}{L^2} \right)$$

Where

D = Displacement of conformal coat

ν = Poisson's ratio

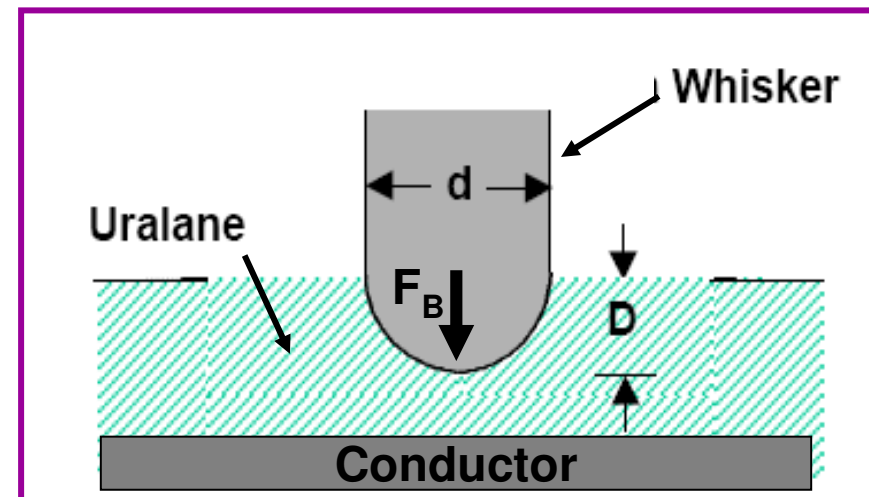
E_{coat} = Young's Modulus of coating

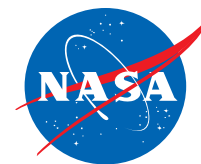
E_W = Young's Modulus of Whisker

d = "Diameter" of whisker

L = Length of whisker

F_B = Euler Buckling Strength
of the whisker



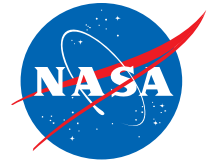


Thank Goodness for Euler Buckling and Conformal Coat on this PWB!!!



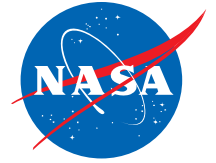
Photo Credit: M&P Failure Analysis Laboratory
The Boeing Company Logistics Depot

Effects of Conformal Coating -- 1



- Numerous sorts of coatings have been tried:
 - Reports of success vary from “none” to “perfect”, sometimes for the same sort of coating.
- NASA GSFC has used Uralane 5750, applied to pre-primed tin-plated surfaces to a thickness of 2 mils (=50 micrometers) +/- 10%:
 - After ~9 years of office ambient storage, these surfaces have whiskered abundantly, but the number of whiskers escaping through the 2 mil thick areas has been zero
- Dr. Thomas Woodrow (Boeing) has studied Urethane (acrylic) coatings, a silicone coating, and Parylene C coating of varying thicknesses up to ~ 4 mils (= 100 micrometers):
 - Some whiskers have penetrated even the thickest coatings when exposed to 25°C / 97% R.H.

Effects of Conformal Coating -- 2



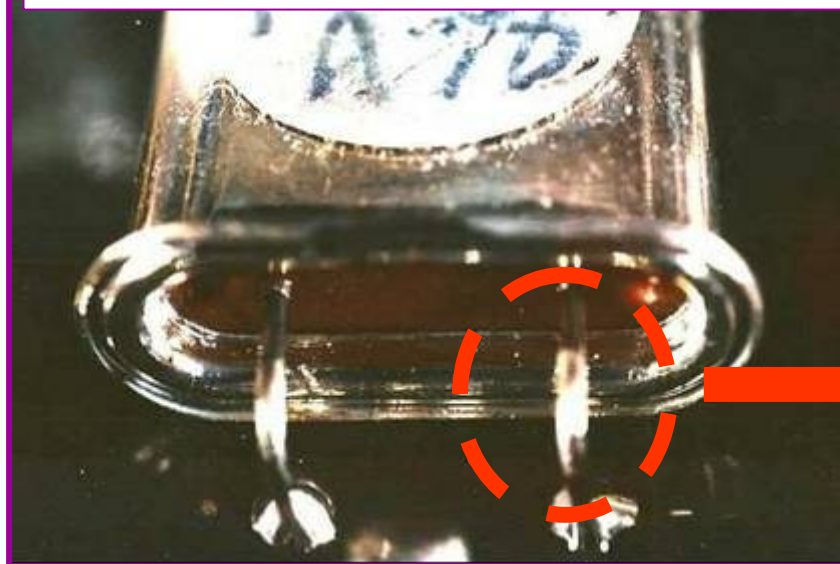
- Conclusion 1:
 - Uralane 5750, applied to at least 2 mils thickness, is a substantial improvement over an uncoated surface.
- Conclusion 2:
 - It is possible to suppose the surface is protected when it is not
 - There can be “weak zones” of thin coating allowing vertical escape
- Conclusion 3:
 - Even “poor” coatings can offer some protection against a whisker coming from a distant source and attempting to contact the protected surface --- long whiskers bend easily (Euler Buckling).
 - Conformal coat protects against a conductive bridge from detached whiskers lying across a pair of conductors

Hot Solder Dip

Benefits & Limitations

Field Failure ONE Year After Assembly

**Crystal with Tin-Plated Kovar Leads
(with Nickel Underplate)**



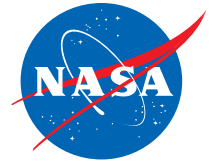
- Leads were Hot Solder Dipped (Sn63Pb37) within 50 mils of Glass Seal BEFORE Mounting to enhance solderability
- Dip was not 100% of leads due to concerns of inducing harm to glass seal

**Tin Whiskers (~60 mils) Grew on
NON-Dipped Region Shorting to Case
Causing Crystal to Malfunction**



- No Whiskers on Hot Solder Dipped Surface
- ABUNDANT whiskers on the Non-Dipped Surface

Contact Information



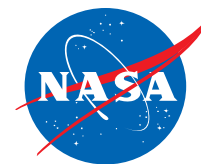
***Jay Brusse
Perot Systems at
NASA Goddard Space Flight Center
Jay.A.Brusse@nasa.gov***

***Work Performed in Support of the
NASA Electronic Parts and Packaging (NEPP) Program***

***Acknowledgment to Dr. Michael Osterman
University of MD – Center for Advanced Life Cycle Engineering (CALCE)***

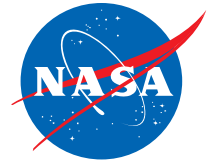
NASA Tin and Other Metal Whisker WWW Site

<http://nepp.nasa.gov/whisker>

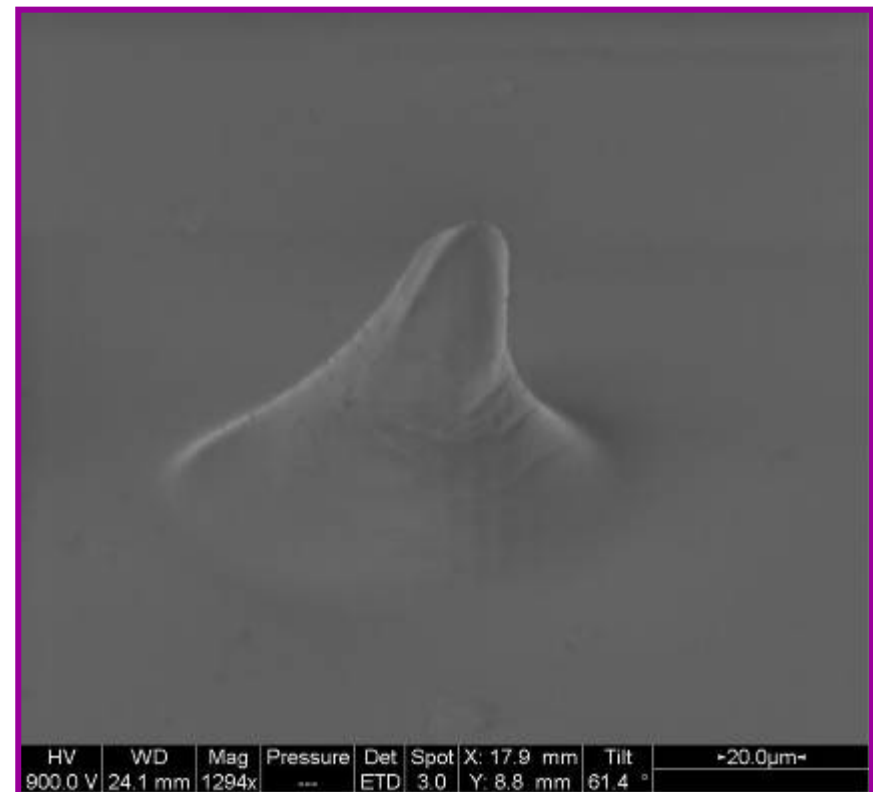
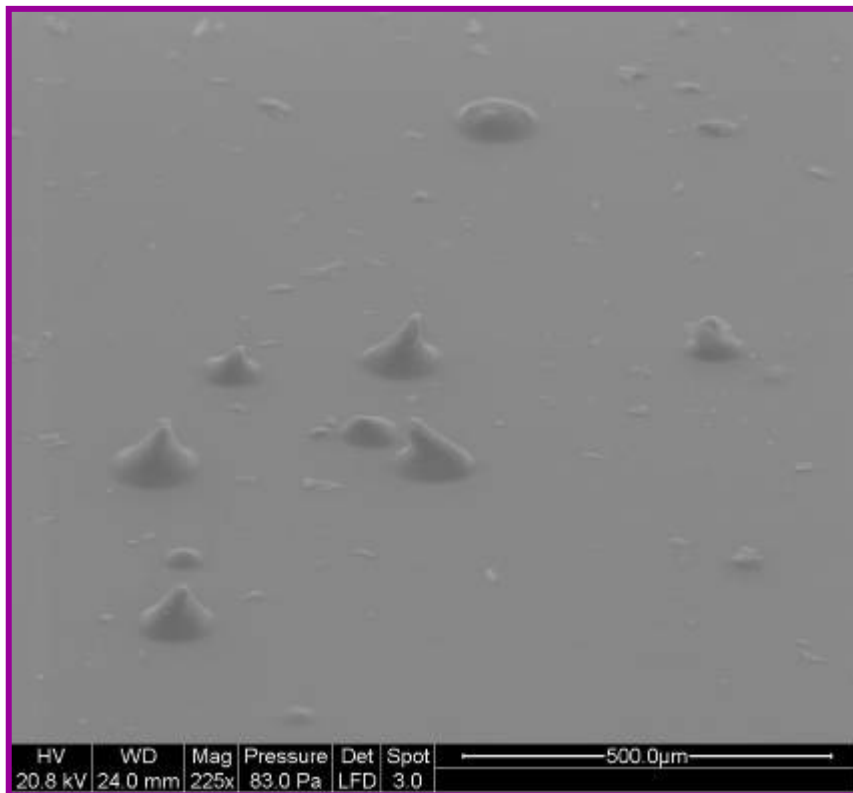


Backup Slides

Tin Whiskers Forming “Circus Tents” in Thin Uralane 5750 Conformal Coat - 9-Years of Office Ambient Storage



Coating Thickness < 0.5 Mil

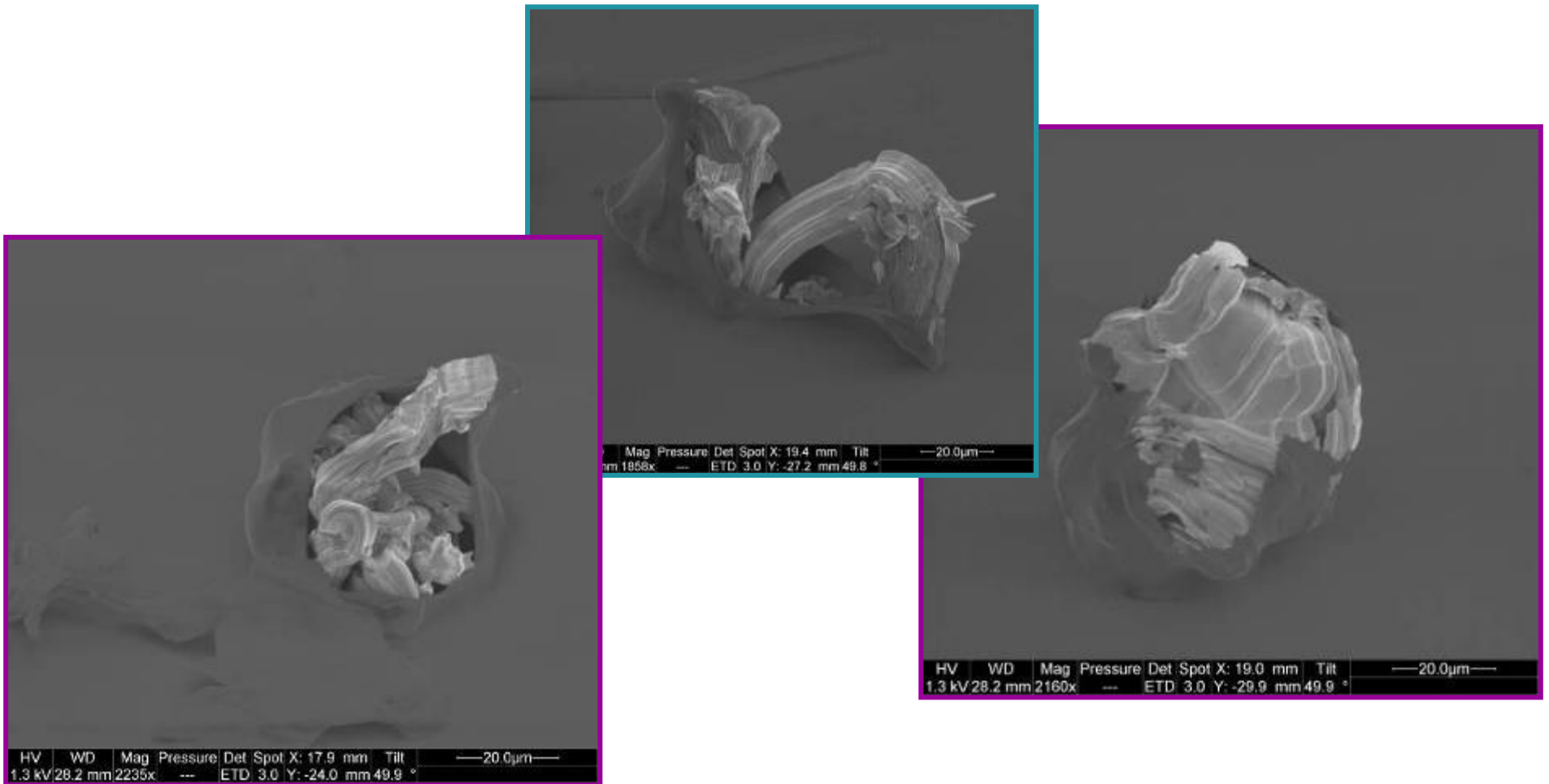
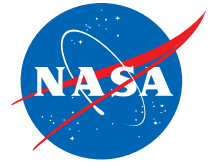


December 5, 2007

Metal Whiskers:
Failure Modes and Mitigation Strategies

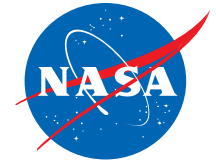
33

Tin Whiskers Rupturing THIN Coating ***~0.1 to 0.2 Mils Uralane 5750 Conformal Coat*** **9-Years of Office Ambient Storage**



December 5, 2007

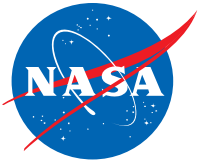
Metal Whiskers:
Failure Modes and Mitigation Strategies



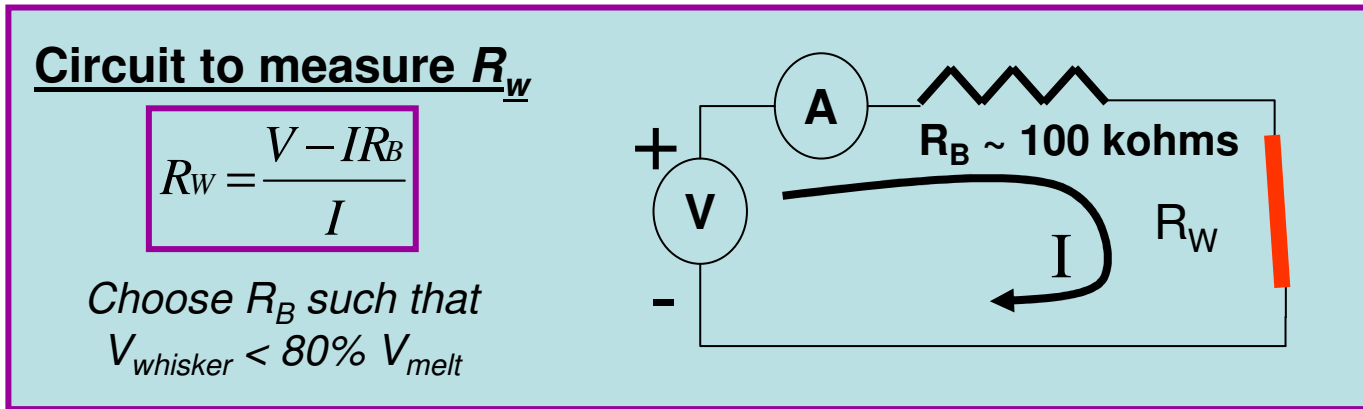
A Few Recent Whisker Experiences

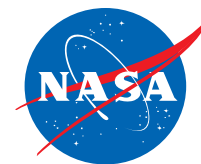
- **Tin Whiskers:**
 - **2005:** Connecticut Nuclear Power Plant shutdown due to Tin whiskers on diodes
 - **2006:** Space Shuttle Transportation System discovers Tin whiskers on card rails: Some 100 to 300 million whiskers were in OV-105's boxes
 - **2006:** SWATCH reports 30% of crystals made using RoHS-compliant tin-copper solder sprouting Tin whiskers. 5% catastrophically shorted within months.
- **Zinc Whiskers:**
 - **2005:** Colorado State Government builds a new “disaster recovery center” after Zinc whiskers crippled its old data center
 - **2005:** 75% of the computer equipment in a particular data center failed due to Zinc whiskers from raised floor tiles. Root cause identified in ~8 months later
 - **2006:** Persistent NAVY weapon system failure confirmed caused by Zinc whiskers
- **Cadmium Whiskers:**
 - **2006:** Spaceflight program observes Cadmium whiskers on electrical switch
 - **2007:** Test chamber feed-thru failure caused by Cadmium whiskers on connector shells

Circuit to Measure Resistance of a Metal Whisker



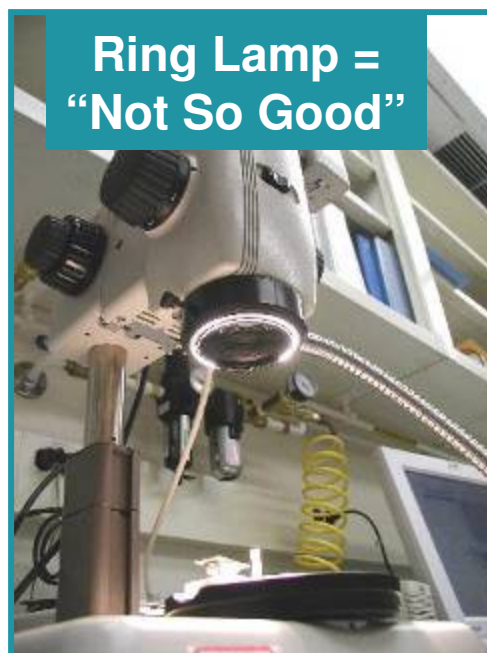
- Use of a simple “Ohmmeter” to measure the resistance of a metal whisker is NOT preferred
 - Ohmmeter may supply $V_{out} < V_{breakdown}$ for the insulating films (oxides, moisture) that form on a metal whisker
 - Ohmmeter may supply $V_{out} > V_{melt}$ causing the whisker to melt before resistance can be measured
- Instead, a variable power supply and a ballast resistor should be used to overcome the above complications
 - Adjust $V_{out} > V_{breakdown}$ of insulating films on whisker
 - When $V_{out} > V_{breakdown}$, R_B quickly drops $V_{whisker} < V_{melt}$





Optical Inspection for Metal Whiskers

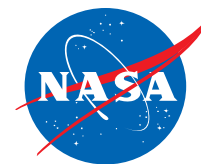
- Basic Equipment:
 - Binocular Microscope
 - Light Source: Flex Lighting PREFERRED over Ring Lamp
- Freedom to tilt sample and/or lighting to illuminate whisker facets is VERY IMPORTANT



December 5, 2007

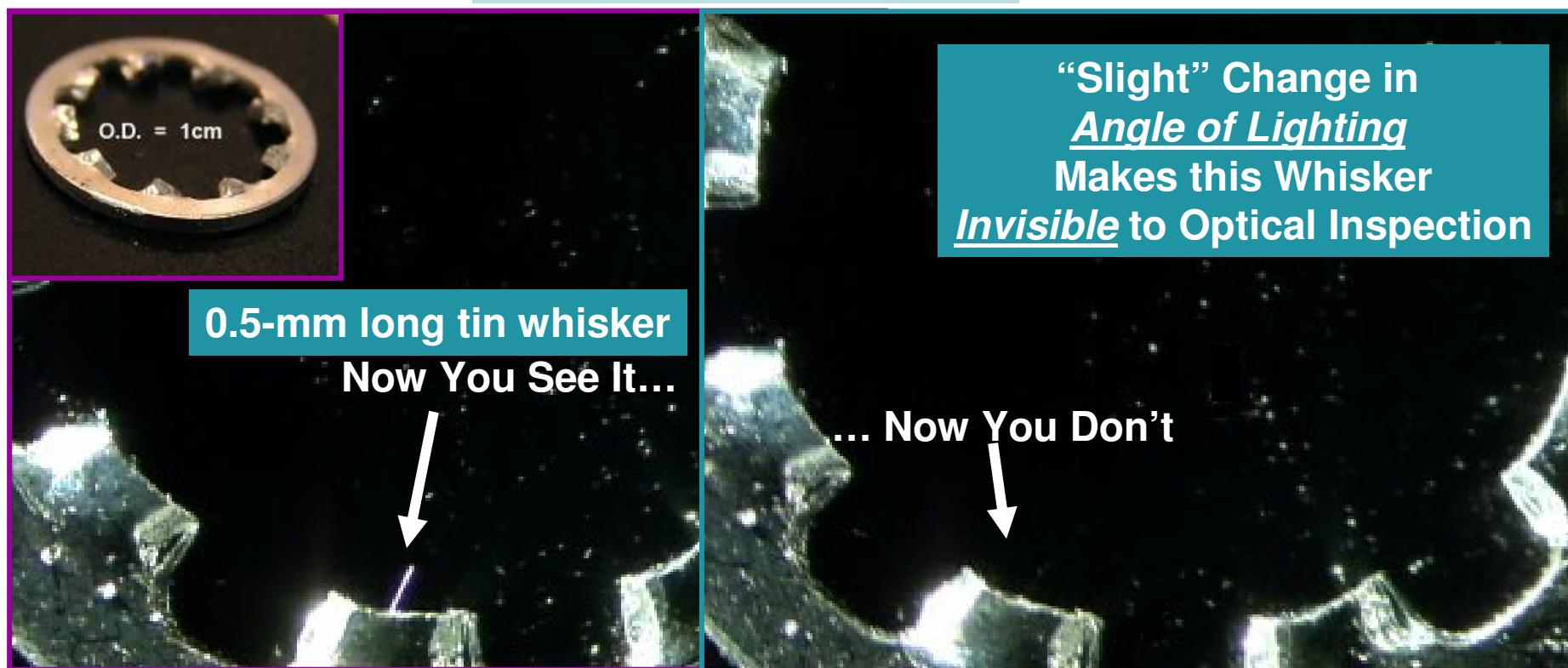
Metal Whiskers:
Failure Modes and Mitigation Strategies

37



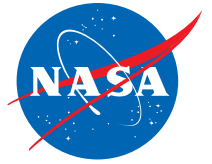
Evidence of “Absence of Whiskers”? (Optical Microscopy)

Tin-Plated Lock Washer



The absence of evidence is NOT evidence of absence

Field Technicians and Failure Analysts Need To Be Acquainted with Metal Whiskers!!!



NASA GSFC has published videos to aid in optical inspection for metal whiskers

<http://nepp.nasa.gov/whisker/video>

Now You See It
Incident Angle Lighting

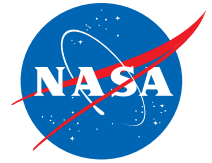


Now You Don't
"Ring Light"



**Small Change in Angle of Lighting
Makes Dramatic Difference
During Optical Inspection**

Why Are Tin, Zinc, Cadmium Still Used?

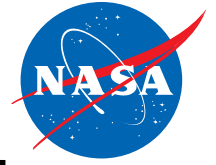


- Not all Tin (or Zinc or Cadmium) surfaces whisker!
 - Rough estimate: 3% to 30% do whisker.
- Not all metal whiskers cause shorts
 - Environment (geometry and electrical potentials matter).
 - Rough estimate: 3% to 30% do short.
- Not all whisker-induced shorts are traced to whiskers
 - They are very hard to see and failure analysis techniques often destroy evidence
 - Rough estimate: 0% to 10% are correctly traced.
- Not all identified whisker adventures are reported
 - Rough estimate: 0% to 3% are reported, once identified
- Hence, we expect between 0.00% and 0.03% of shorting problems caused by these coatings to be reported
 - While some 0.1% to 10% of these coatings are actually causing shorts.
 - With such a few public cases, many say “What, me worry?”
- Whiskering is dramatically inhibited when 0.5% (or more) lead (Pb) is added to Tin coatings: the shorting rate then approaches zero
 - This has been the case for the Hi-Rel community
 - But Pb use is being restricted by international legislation, and so the shorting rate may jump to 10% from zero ==> **SWATCH GROUP** <==

"The Five Stages of Metal Whisker Grief"

By Henning Leidecker

Adapted from Elisabeth Kubler-Ross in her book "On Death and Dying",
Macmillan Publishing Company, 1969



Denial

"Metal whiskers?!? We ain't got no stinkin' whiskers! I don't even think metal whiskers exist! I KNOW we don't have any!"

Anger

"You say we got whiskers, I rip your \$%#@ lungs out! Who put them there --- I'll murderize him! I'll tear him into pieces so small, they'll fit under one of those *^&\$#% whiskers!"

Bargaining

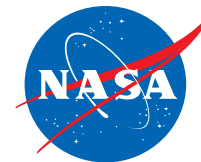
"We have metal whiskers? But they are so small. And you have only seen a few of them. How could a few small things possibly be a problem to our power supplies and equipment? These few whiskers should be easy to clean up."

Depression

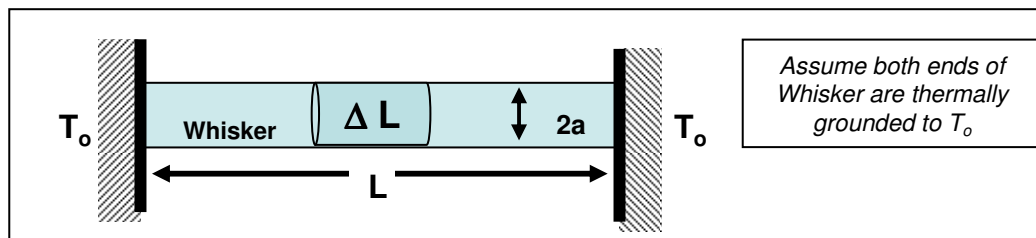
"Dang. Doomed. Close the shop --- we are out of business. Of all the miserable bit joints in all the world, metal whiskers had to come into mine... I'm retiring from here... Going to open a 'Squat & Gobble' on the Keys. "

Acceptance

"Metal whiskers. How about that? Who knew? Well, clean what you can. Put in the particle filters, and schedule periodic checks of what the debris collectors find. Ensure that all the warranties and service plans are up to date. On with life."



Derivation of Melting Current of a Metal Whisker in Vacuum



$$\frac{du}{dt} + \Phi = source$$

du/dt

+

Φ

=

source

$$u = C \cdot T \quad c = \frac{C}{V}$$

$$u = \left(\frac{C}{V} \right) \cdot V \cdot T = c \cdot V \cdot T$$

$$u = c \cdot \Delta L \cdot A \cdot T$$

$$\boxed{\frac{du}{dt} = c \cdot \Delta L \cdot A \cdot \frac{\partial T}{\partial t}}$$

$$\Phi = \left(\frac{\partial J}{\partial x} \right) \cdot \Delta L \cdot A$$

Convection loss = 0 for vacuum
Neglect radiation loss

$$J = -k_T \cdot \frac{\partial T}{\partial x}$$

$$\frac{\partial J}{\partial x} = -k_T \cdot \frac{\partial^2 T}{\partial x^2}$$

$$\Phi = -k_T \cdot \left(\frac{\partial^2 T}{\partial x^2} \right) \cdot \Delta L \cdot A$$

$$k_T = \frac{L_z \cdot T}{\rho}$$

$$\boxed{\Phi = -\frac{L_z \cdot T}{\rho} \left(\frac{\partial^2 T}{\partial x^2} \right) \cdot \Delta L \cdot A}$$

$$source = I^2 \cdot R$$

$$I = J_e \cdot A \quad R = \frac{\rho \cdot \Delta L}{A}$$

$$source = (J_e^2 \cdot A^2) \cdot \left(\frac{\rho \cdot \Delta L}{A} \right)$$

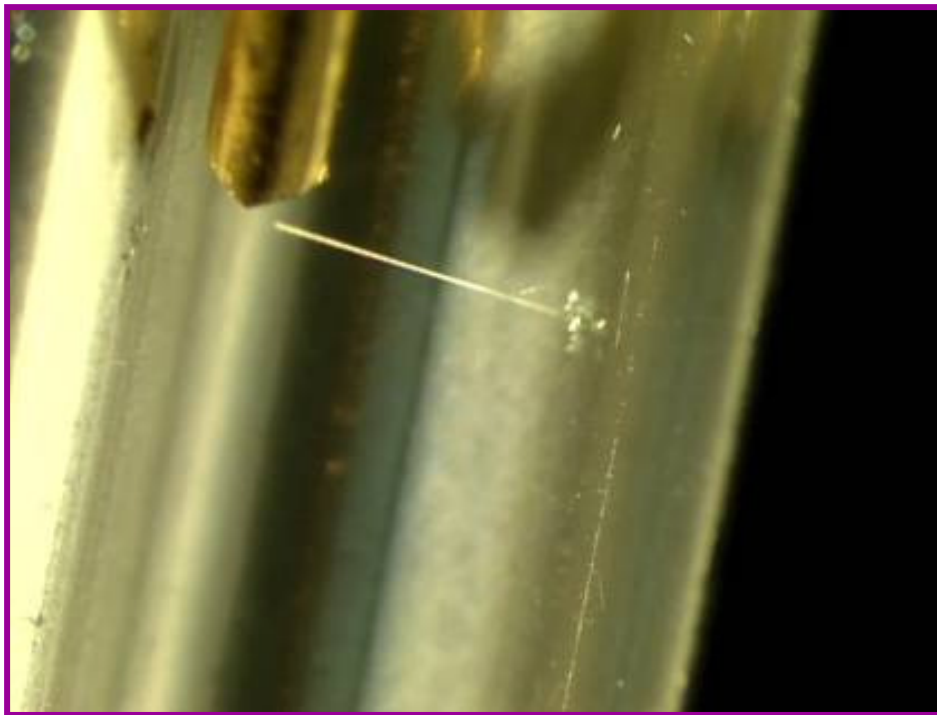
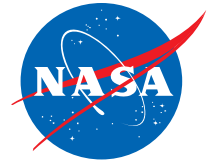
$$\boxed{source = (J_e^2 \cdot A) \cdot \rho \cdot \Delta L}$$

$$\left[c \cdot \Delta L \cdot A \cdot \frac{\partial T}{\partial t} \right] - \left[\frac{L_z \cdot T}{\rho} \left(\frac{\partial^2 T}{\partial x^2} \right) \cdot \Delta L \cdot A \right] = J^2 \cdot \rho \cdot \Delta L \cdot A$$

$$\left[c \cdot \frac{\partial T}{\partial t} \right] - \left[\frac{L_z \cdot T}{\rho} \left(\frac{\partial^2 T}{\partial x^2} \right) \right] = J^2 \cdot \rho$$

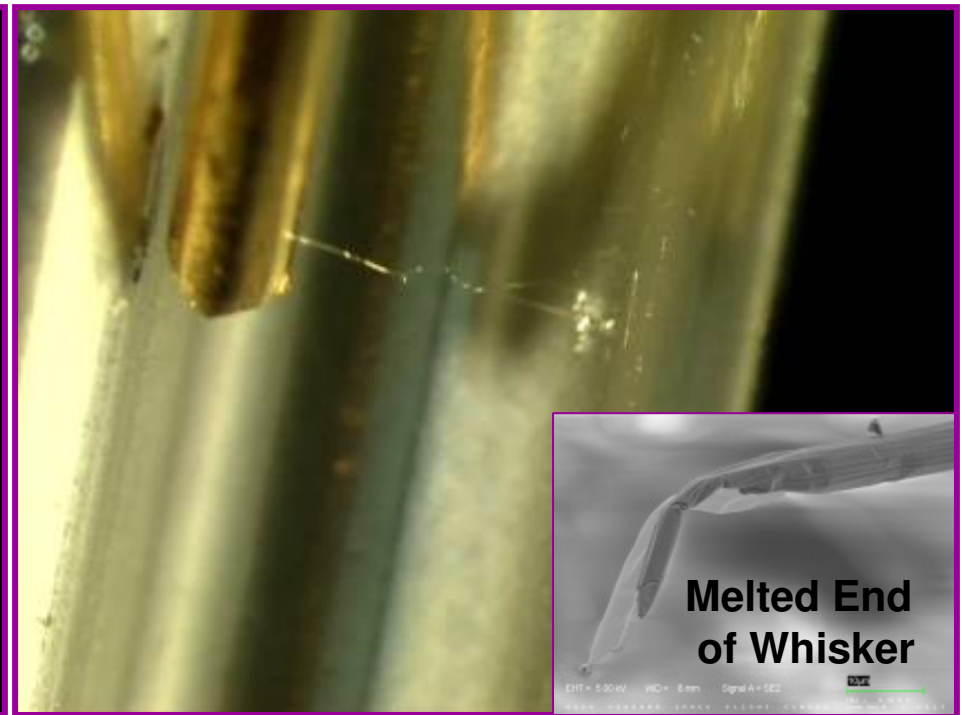
$$I_{melt,vac} = \left[\frac{2\sqrt{L_z T_0}}{R_0} \right] \cos^{-1} \left(\frac{T_0}{T_{melt}} \right)$$

An Example of “Melting” a Tin Whisker



Before Contact

1. Gold-Plated Test Probe has +3 Volts Relative to Tin Whisker



After Contact

1. Tip of whisker micro-welds to gold test probe
2. Whisker melts mid-length
3. Small section of whisker root remains attached to substrate